

# **Learning logic programs by discovering where not to search**

**Andrew Cropper and Céline Hocquette**



Engineering and  
Physical Sciences  
Research Council



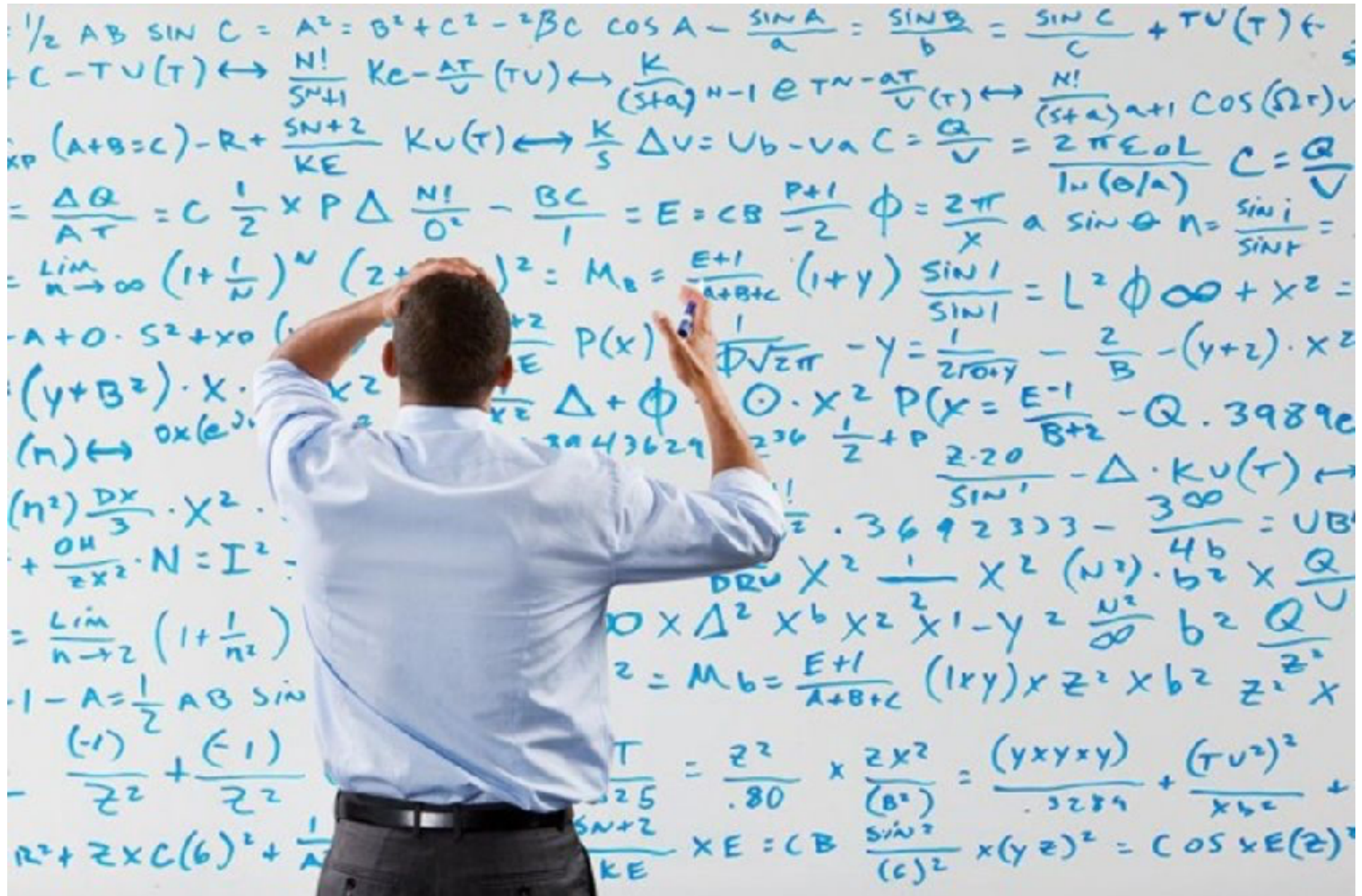
# **What is this talk about?**

- **A simple approach to improve the learning performance of an ILP system**

# **What is this talk about?**

- **A simple approach to improve the learning performance of an ILP system**
- **The idea is to discover where not to search before searching for a solution**

# No technical details



# **Inductive logic programming**

# Inductive logic programming

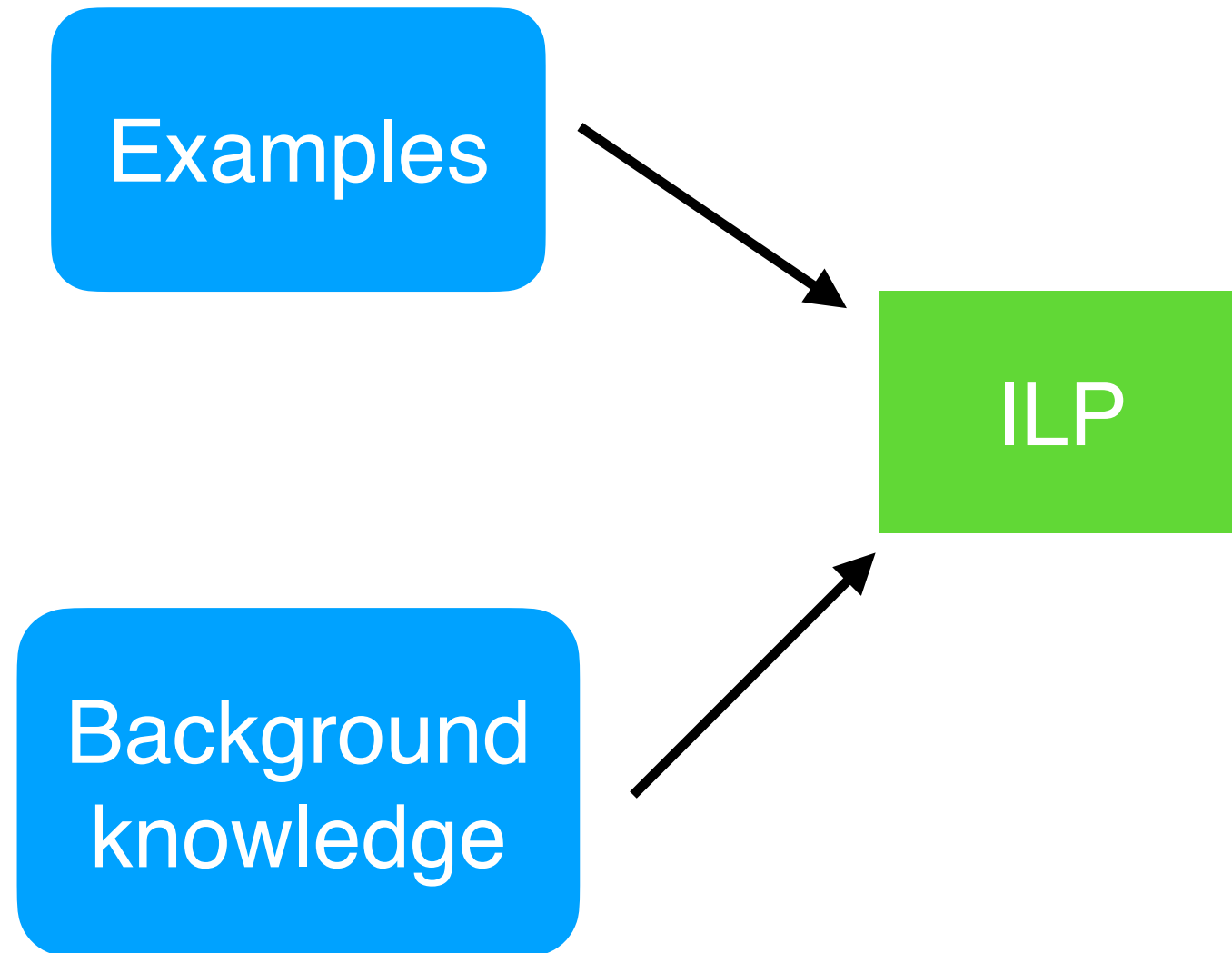
Examples

# Inductive logic programming

Examples

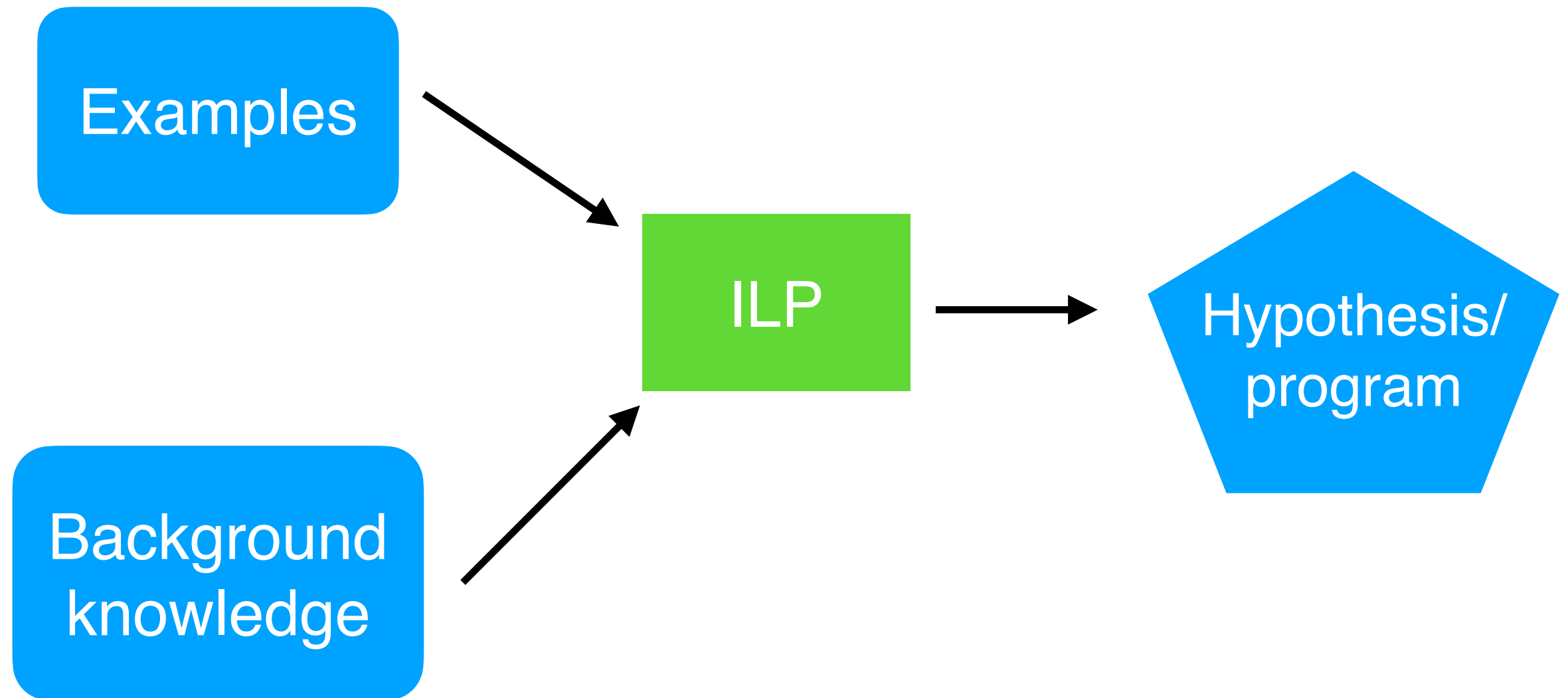
Background  
knowledge

# Inductive logic programming





# Inductive logic programming



# Examples

input	output
dog	g
sheep	p
chicken	n

# Examples

input	output
dog	g
sheep	p
chicken	n

representation
last(dog,g)
last(sheep,p)
last(chicken,n)

# BK

```
head([H|_],H).  
tail([_|T],T).  
empty(A).
```

# Hypothesis

`last(A,B):-tail(A,C),empty(C),head(A,B).`  
`last(A,B):-tail(A,C),f(C,B).`

# Motivation

**Very large hypothesis spaces**

# Idea

**Can we discover where not to search before searching for a solution?**

BK

<i>head(ijcai,i)</i>	<i>tail(ijcai,jcai)</i>	<i>even(2)</i>
<i>head(ecai,e)</i>	<i>tail(ecai,cai)</i>	<i>even(4)</i>
<i>head(cai,c)</i>	<i>tail(jcai,cai)</i>	<i>odd(1)</i>
<i>tail(ai,i)</i>	<i>tail(cai,ai)</i>	<i>odd(3)</i>



$$\begin{aligned}
r_1 &= h \leftarrow \text{tail}(A,A) \\
r_2 &= h \leftarrow \text{tail}(A,B), \text{tail}(B,A) \\
r_3 &= h \leftarrow \text{tail}(A,B), \text{tail}(B,C), \text{tail}(A,C) \\
r_4 &= h \leftarrow \text{tail}(A,A), \text{head}(A,B), \text{odd}(B) \\
r_5 &= h \leftarrow \text{head}(A,B), \text{odd}(B), \text{even}(B)
\end{aligned}$$

<i>head(ijcai,i)</i>	<i>tail(ijcai,jcai)</i>	<i>even(2)</i>
<i>head(ecai,e)</i>	<i>tail(ecai,cai)</i>	<i>even(4)</i>
<i>head(cai,c)</i>	<i>tail(jcai,cai)</i>	<i>odd(1)</i>
<i>tail(ai,i)</i>	<i>tail(cai,ai)</i>	<i>odd(3)</i>

tail is not reflexive

$$\begin{aligned} & \text{--- } r_1 = h \leftarrow \text{tail}(A, A) \text{ ---} \\ & r_2 = h \leftarrow \text{tail}(A, B), \text{tail}(B, A) \\ & r_3 = h \leftarrow \text{tail}(A, B), \text{tail}(B, C), \text{tail}(A, C) \\ & \text{--- } r_4 = h \leftarrow \text{tail}(A, A), \text{head}(A, B), \text{odd}(B) \text{ ---} \\ & r_5 = h \leftarrow \text{head}(A, B), \text{odd}(B), \text{even}(B) \end{aligned}$$

<i>head(ijcai, i)</i>	<i>tail(ijcai, jcai)</i>	<i>even(2)</i>
<i>head(ecai, e)</i>	<i>tail(ecai, cai)</i>	<i>even(4)</i>
<i>head(cai, c)</i>	<i>tail(jcai, cai)</i>	<i>odd(1)</i>
<i>tail(ai, i)</i>	<i>tail(cai, ai)</i>	<i>odd(3)</i>

tail is not symmetric

$$\begin{aligned} & \text{--- } r_1 = h \leftarrow \text{tail}(A, A) \text{ ---} \\ & \text{--- } r_2 = h \leftarrow \text{tail}(A, B), \text{tail}(B, A) \text{ ---} \\ & r_3 = h \leftarrow \text{tail}(A, B), \text{tail}(B, C), \text{tail}(A, C) \\ & \text{--- } r_4 = h \leftarrow \text{tail}(A, A), \text{head}(A, B), \text{odd}(B) \text{ ---} \\ & r_5 = h \leftarrow \text{head}(A, B), \text{odd}(B), \text{even}(B) \end{aligned}$$

<i>head(ijcai, i)</i>	<i>tail(ijcai, jcai)</i>	<i>even(2)</i>
<i>head(ecai, e)</i>	<i>tail(ecai, cai)</i>	<i>even(4)</i>
<i>head(cai, c)</i>	<i>tail(jcai, cai)</i>	<i>odd(1)</i>
<i>tail(ai, i)</i>	<i>tail(cai, ai)</i>	<i>odd(3)</i>

tail is not transitive

$$\begin{aligned} & \text{--- } r_1 = h \leftarrow \text{tail}(A, A) \\ & \text{--- } r_2 = h \leftarrow \text{tail}(A, B), \text{tail}(B, A) \\ & \text{--- } r_3 = h \leftarrow \text{tail}(A, B), \text{tail}(B, C), \text{tail}(A, C) \\ & \text{--- } r_4 = h \leftarrow \text{tail}(A, A), \text{head}(A, B), \text{odd}(B) \\ & \quad r_5 = h \leftarrow \text{head}(A, B), \text{odd}(B), \text{even}(B) \end{aligned}$$

$\text{head}(\text{ijcai}, i)$	$\text{tail}(\text{ijcai}, \text{jcai})$	$\text{even}(2)$
$\text{head}(\text{ecai}, e)$	$\text{tail}(\text{ecai}, \text{cai})$	$\text{even}(4)$
$\text{head}(\text{cai}, c)$	$\text{tail}(\text{jcai}, \text{cai})$	$\text{odd}(1)$
$\text{tail}(\text{ai}, i)$	$\text{tail}(\text{cai}, \text{ai})$	$\text{odd}(3)$

odd(B) and even(B) are unsatisfiable

~~$r_1 = h \leftarrow \text{tail}(A, A)$~~   
 ~~$r_2 = h \leftarrow \text{tail}(A, B), \text{tail}(B, A)$~~   
 ~~$r_3 = h \leftarrow \text{tail}(A, B), \text{tail}(B, C), \text{tail}(A, C)$~~   
 ~~$r_4 = h \leftarrow \text{tail}(A, A), \text{head}(A, B), \text{odd}(B)$~~   
 ~~$r_5 = h \leftarrow \text{head}(A, B), \text{odd}(B), \text{even}(B)$~~

$\text{head}(\text{ijcai}, i)$	$\text{tail}(\text{ijcai}, \text{jcai})$	$\text{even}(2)$
$\text{head}(\text{ecai}, e)$	$\text{tail}(\text{ecai}, \text{cai})$	$\text{even}(4)$
$\text{head}(\text{cai}, c)$	$\text{tail}(\text{jcai}, \text{cai})$	$\text{odd}(1)$
$\text{tail}(\text{ai}, i)$	$\text{tail}(\text{cai}, \text{ai})$	$\text{odd}(3)$

**We have eliminated hypotheses before even  
considering the examples**

**How?**

# How?

- 1. Preprocess the BK to discover constraints**



# How?

- 1. Preprocess the BK to discover constraints**
- 2. Use the constraints to bootstrap a constraint-driven ILP system**

# Constraint discovery

Name	Property	Constraint	Example
Irreflexive	$\neg p(A,A)$	$\leftarrow p(A,A)$	$\leftarrow \text{brother}(A,A)$
Antitransitive	$p(A,B), p(B,C) \rightarrow \neg p(A,C)$	$\leftarrow p(A,B), p(B,C), p(A,C)$	$\leftarrow \text{succ}(A,B), \text{succ}(B,C), \text{succ}(A,C)$
Antitriangular	$p(A,B), p(B,C) \rightarrow \neg p(C,A)$	$\leftarrow p(A,B), p(B,C), p(C,A)$	$\leftarrow \text{tail}(A,B), \text{tail}(B,C), \text{tail}(C,A)$
Injective	$p(A,B), p(C,B) \rightarrow A=C$	$\leftarrow p(A,B), p(C,B), A \neq C$	$\leftarrow \text{succ}(A,B), \text{succ}(C,B), A \neq C$
Functional	$p(A,B), p(A,C) \rightarrow B=C$	$\leftarrow p(A,B), p(A,C), B \neq C$	$\leftarrow \text{length}(A,B), \text{length}(A,C), B \neq C$
Asymmetric	$p(A,B) \rightarrow \neg p(B,A)$	$\leftarrow p(A,B), p(B,A)$	$\leftarrow \text{mother}(A,B), \text{mother}(B,A)$
Exclusive	$p(A) \rightarrow \neg q(A)$	$\leftarrow p(A), q(A)$	$\leftarrow \text{odd}(A), \text{even}(A)$

# Constraint discovery

Name	Property	Constraint	Example
Irreflexive	$\neg p(A,A)$	$\leftarrow p(A,A)$	$\leftarrow \text{brother}(A,A)$
Antitransitive	$p(A,B), p(B,C) \rightarrow \neg p(A,C)$	$\leftarrow p(A,B), p(B,C), p(A,C)$	$\leftarrow \text{succ}(A,B), \text{succ}(B,C), \text{succ}(A,C)$
Antitriangular	$p(A,B), p(B,C) \rightarrow \neg p(C,A)$	$\leftarrow p(A,B), p(B,C), p(C,A)$	$\leftarrow \text{tail}(A,B), \text{tail}(B,C), \text{tail}(C,A)$
Injective	$p(A,B), p(C,B) \rightarrow A=C$	$\leftarrow p(A,B), p(C,B), A \neq C$	$\leftarrow \text{succ}(A,B), \text{succ}(C,B), A \neq C$
Functional	$p(A,B), p(A,C) \rightarrow B=C$	$\leftarrow p(A,B), p(A,C), B \neq C$	$\leftarrow \text{length}(A,B), \text{length}(A,C), B \neq C$
Asymmetric	$p(A,B) \rightarrow \neg p(B,A)$	$\leftarrow p(A,B), p(B,A)$	$\leftarrow \text{mother}(A,B), \text{mother}(B,A)$
Exclusive	$p(A) \rightarrow \neg q(A)$	$\leftarrow p(A), q(A)$	$\leftarrow \text{odd}(A), \text{even}(A)$

**Use ASP programs to discover the constraints**

# Functional property

$$p(A,B), p(A,C) \rightarrow B=C$$

# Functional constraint

$\leftarrow p(A,B), p(A,C), B \neq C$

# Functional example

←  $length(A,B), length(A,C), B \neq C$

# Bootstrapping

**We use the constraints to bootstrap Popper, a constraint-driven ILP system.**

**Does it work?**



**Does it work?**

**How long does BK preprocessing take?**

<b>Domain</b>	<b>Time</b>
<i>trains</i>	$0.22 \pm 0.00$
<i>zendo</i>	$0.03 \pm 0.00$
<i>imdb</i>	$0.02 \pm 0.00$
<i>krk</i>	$0.10 \pm 0.00$
<i>rps</i>	$0.02 \pm 0.00$
<i>centipede</i>	$0.02 \pm 0.00$
<i>md</i>	$0.01 \pm 0.00$
<i>buttons</i>	$0.02 \pm 0.00$
<i>attrition</i>	$0.01 \pm 0.00$
<i>coins</i>	$0.03 \pm 0.00$
<i>synthesis</i>	$4.00 \pm 0.40$

# **Does it work?**

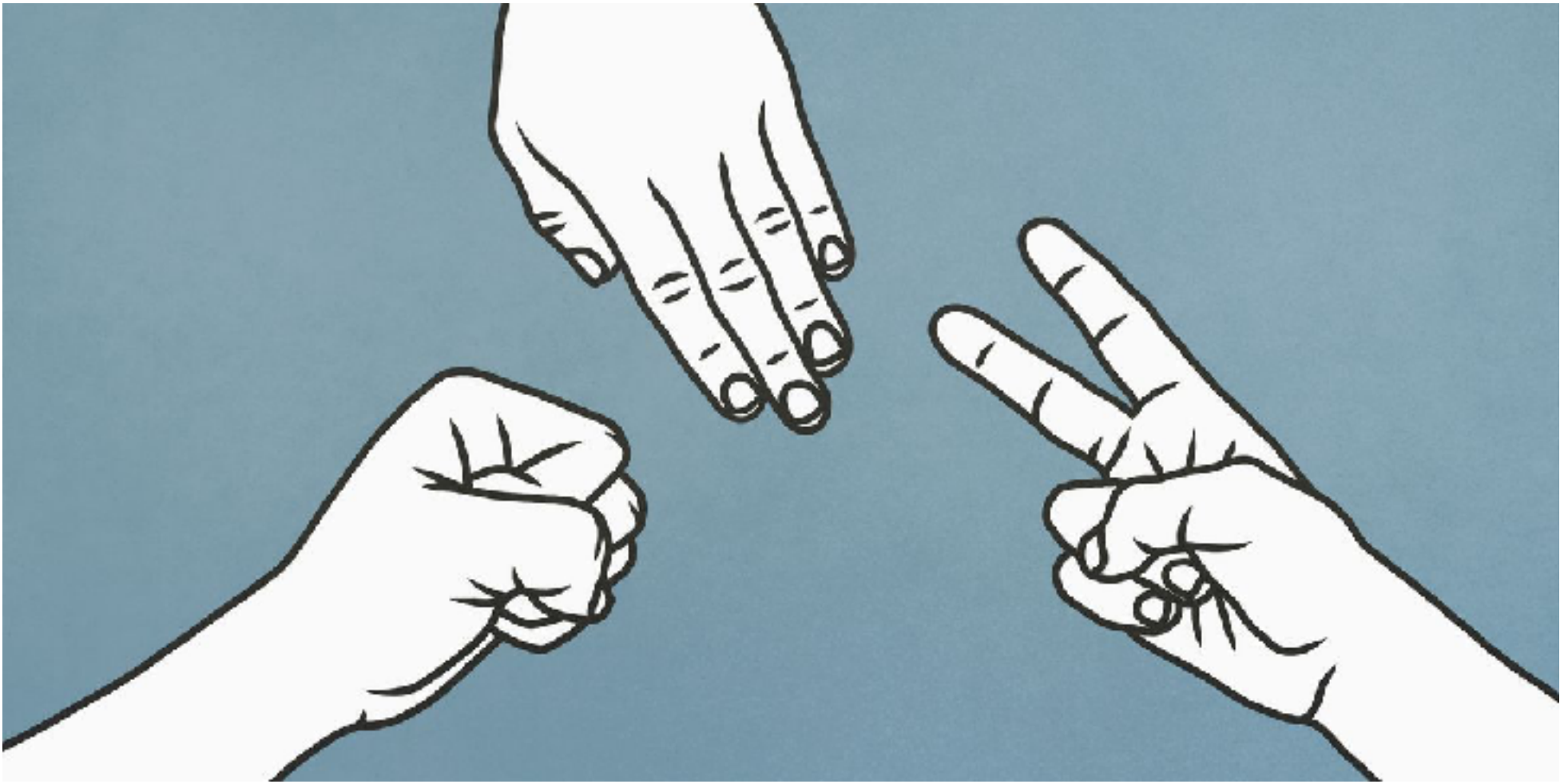
**Q1. Can BK constraint discovery reduce learning times?**

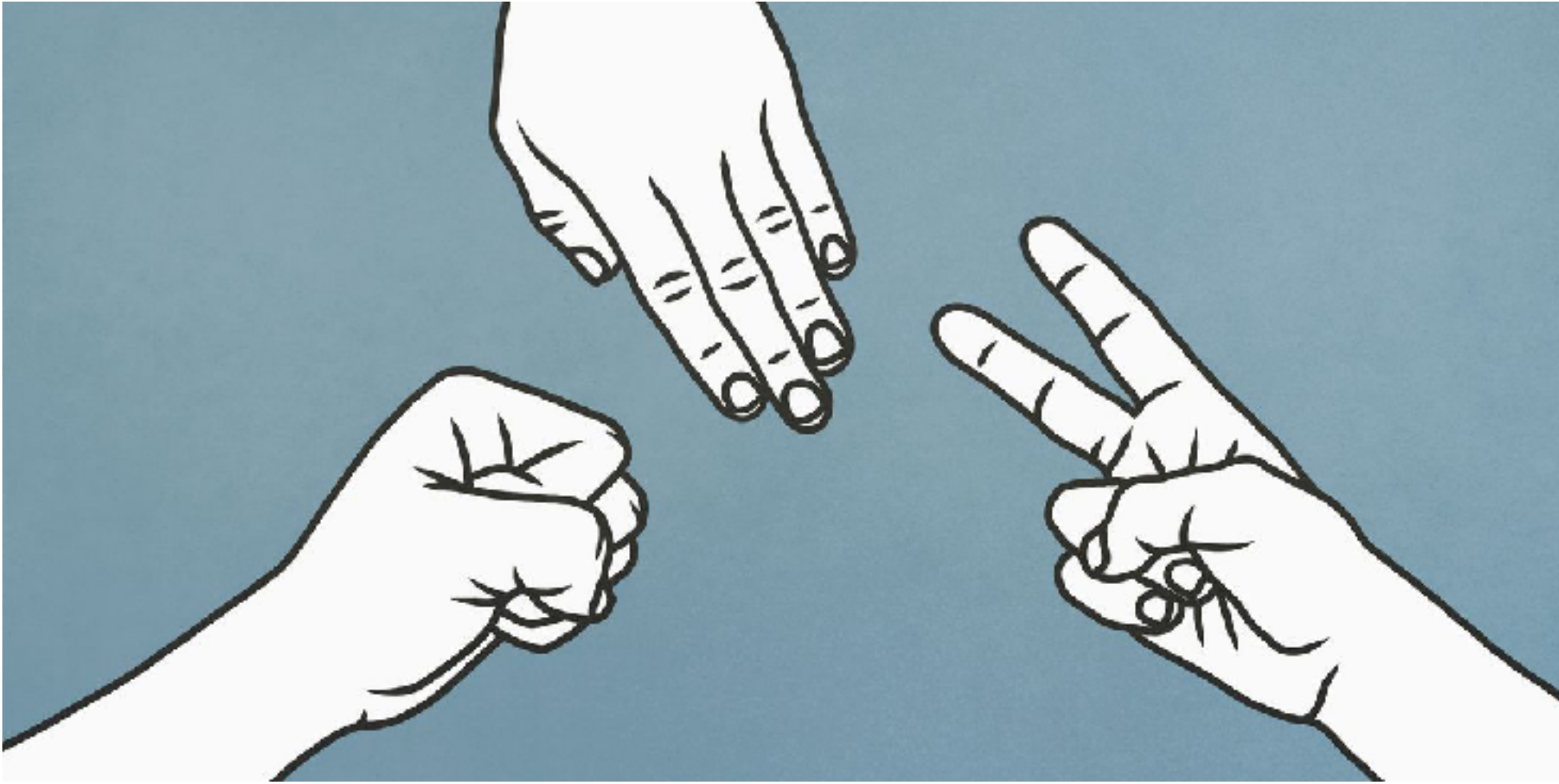
**Does it work?**

**Yes!**

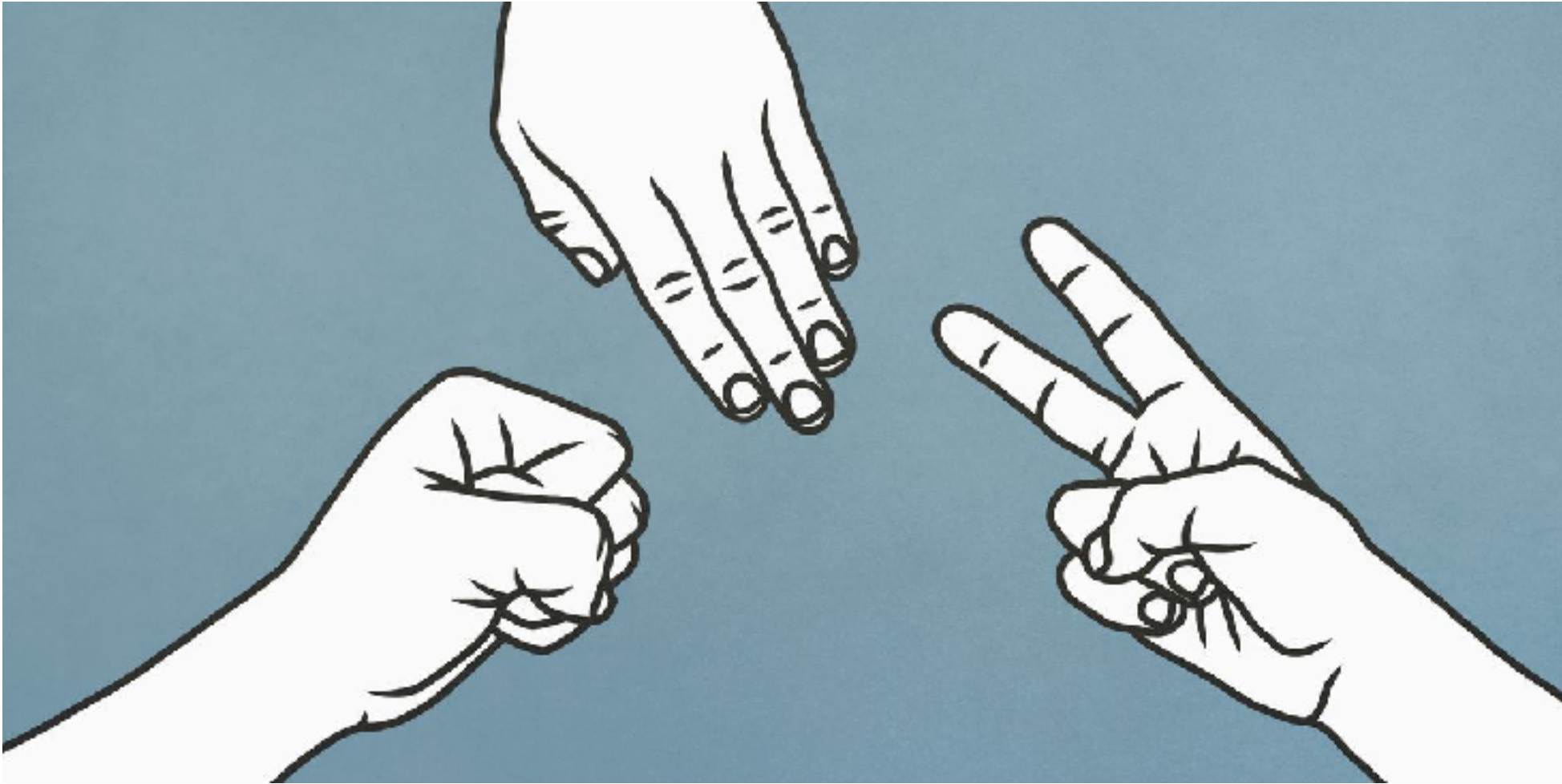


<b>Task</b>	<b>POPPER</b>	<b>DISCO</b>	<b>Change</b>
<i>trains1</i>	$5 \pm 0.1$	$4 \pm 0.1$	<b>-20%</b>
<i>trains2</i>	$5 \pm 0.2$	$4 \pm 0.3$	<b>-20%</b>
<i>trains3</i>	$27 \pm 0.8$	$22 \pm 0.6$	<b>-18%</b>
<i>trains4</i>	$24 \pm 0.8$	$20 \pm 0.5$	<b>-16%</b>
<i>zendo1</i>	$8 \pm 2$	$6 \pm 1$	<b>-25%</b>
<i>zendo2</i>	$32 \pm 2$	$31 \pm 2$	<b>-3%</b>
<i>zendo3</i>	$33 \pm 2$	$31 \pm 1$	<b>-6%</b>
<i>zendo4</i>	$24 \pm 3$	$24 \pm 3$	<b>0%</b>
<i>imdb1</i>	$1 \pm 0$	$1 \pm 0$	<b>0%</b>
<i>imdb2</i>	$2 \pm 0.1$	$2 \pm 0$	<b>0%</b>
<i>imdb3</i>	$366 \pm 23$	$287 \pm 17$	<b>-21%</b>
<i>krk</i>	$48 \pm 6$	$9 \pm 0.6$	<b>-81%</b>
<i>rps</i>	$37 \pm 1$	$6 \pm 0.2$	<b>-83%</b>
<i>centipede</i>	$47 \pm 2$	$9 \pm 0.2$	<b>-80%</b>
<i>md</i>	$142 \pm 7$	$13 \pm 0.4$	<b>-90%</b>
<i>buttons</i>	$686 \pm 109$	$25 \pm 1$	<b>-96%</b>
<i>attrition</i>	$410 \pm 20$	$57 \pm 2$	<b>-86%</b>
<i>coins</i>	$496 \pm 19$	$345 \pm 18$	<b>-30%</b>
<i>buttons-goal</i>	$11 \pm 0.2$	$5 \pm 0.1$	<b>-54%</b>
<i>coins-goal</i>	$122 \pm 6$	$76 \pm 2$	<b>-37%</b>
<i>dropk</i>	$4 \pm 0.3$	$3 \pm 0.2$	<b>-25%</b>
<i>droplast</i>	$41 \pm 3$	$23 \pm 2$	<b>-43%</b>
<i>evens</i>	$33 \pm 7$	$9 \pm 1$	<b>-72%</b>
<i>finddup</i>	$51 \pm 8$	$32 \pm 4$	<b>-37%</b>
<i>last</i>	$4 \pm 0.4$	$3 \pm 0.2$	<b>-25%</b>
<i>len</i>	$31 \pm 5$	$16 \pm 2$	<b>-48%</b>
<i>sorted</i>	$74 \pm 5$	$23 \pm 1$	<b>-68%</b>
<i>sumlist</i>	$554 \pm 122$	$320 \pm 40$	<b>-42%</b>





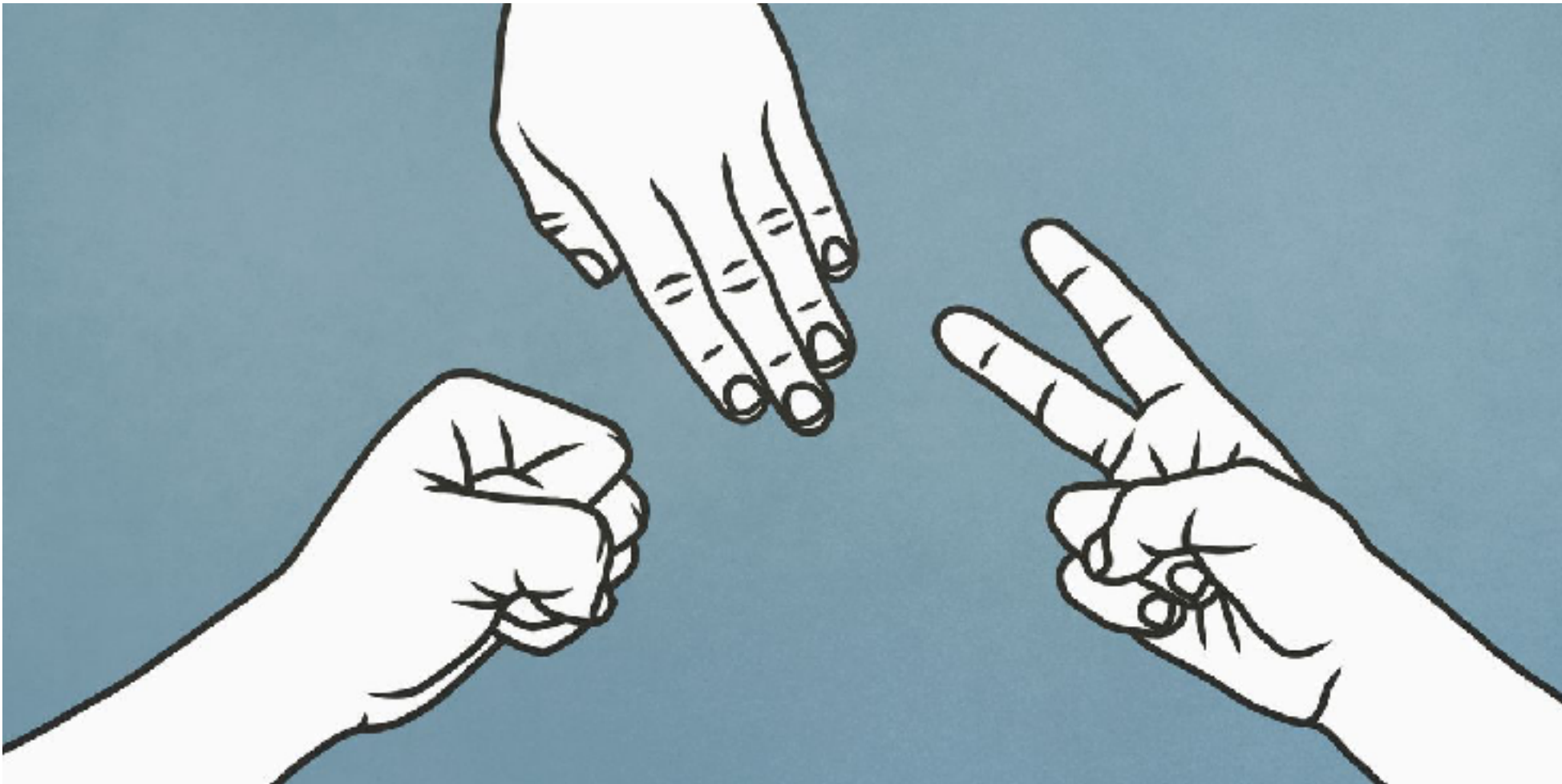
**succ/2**



**$\text{succ}/2$**

irreflexive, injective, functional, antitransitive, antitriangular, and  
asymmetric





**succ/2**

irreflexive, injective, functional, antitransitive, antitriangular, and  
asymmetric

The resulting constraints reduce the number of rules in the  
hypothesis space from **1,189,916** to **70,270**

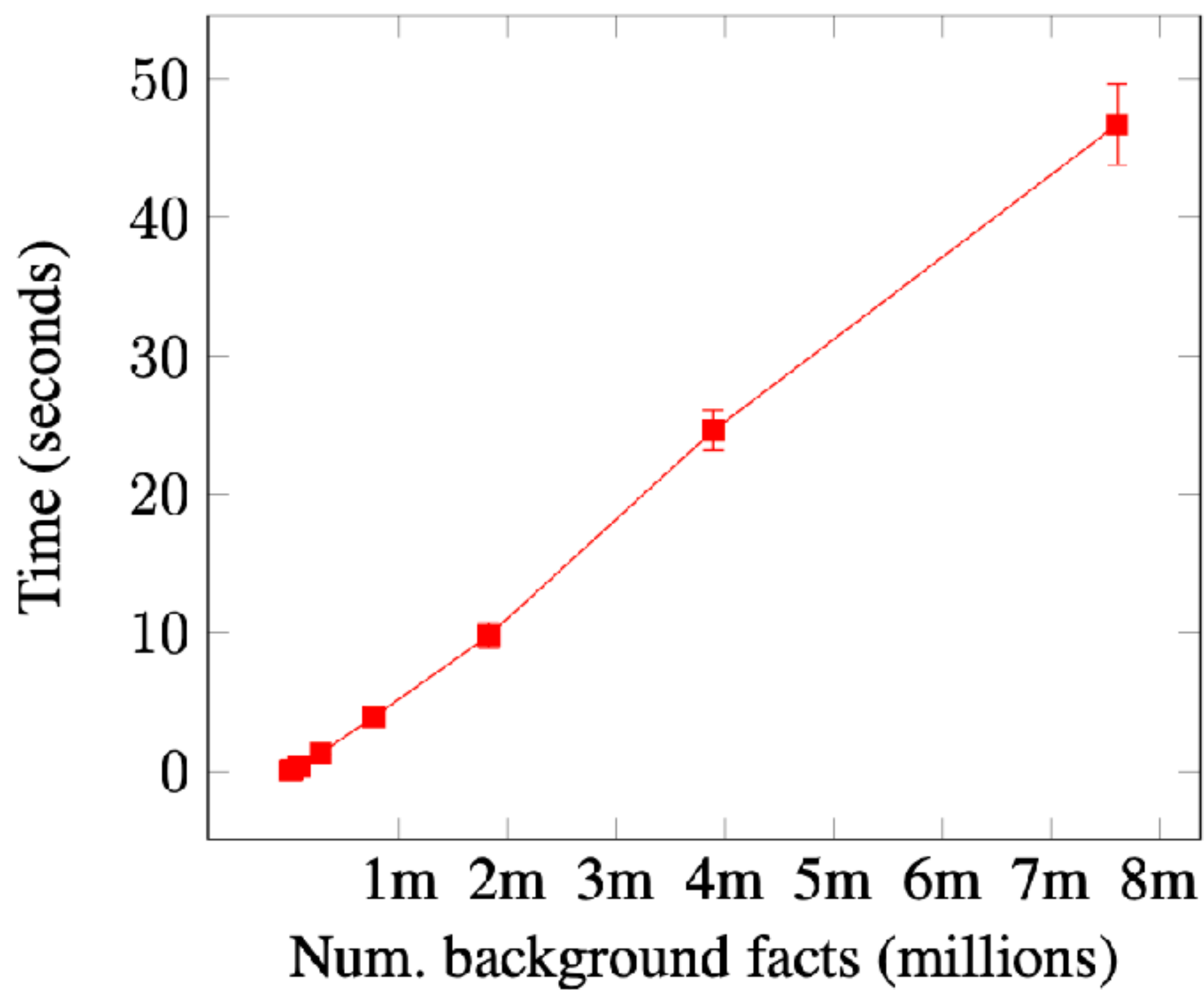
# **Does it work?**

**Q2. What effect does BK constraint discovery have on learning times given larger hypothesis spaces?**

<b>Size</b>	<b>POPPER</b>	<b>DISCO</b>	<b>Change</b>
5	$12 \pm 0.9$	$3 \pm 0.3$	<b>-75%</b>
6	$113 \pm 2$	$10 \pm 0.1$	<b>-91%</b>
7	$864 \pm 156$	$23 \pm 0.9$	<b>-97%</b>
8	<i>timeout</i>	$47 \pm 2$	<b>-96%</b>
9	<i>timeout</i>	$48 \pm 3$	<b>-96%</b>
10	<i>timeout</i>	$52 \pm 0.1$	<b>-95%</b>

**Does it work?**

**Q3. How long does BK constraint discovery take  
given larger BK?**



# **Why does this idea work?**

**We only need one counter-example to eliminate a property**

**Why does this idea work?**

Our properties are small

# Why care?

Simple, general, and performs well



# Why care?

Simple, general, and performs well

**Can easily be made better**

**What can be improved?**

Assume a finite domain (Datalog programs)

**What can be improved?**

Assume given properties to discover

**Questions?**

**Poster ID 191**

**<https://github.com/logic-and-learning-lab/Popper>**