Learning logic programs with Popper

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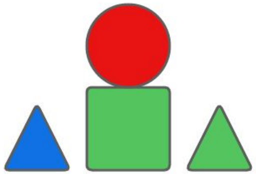
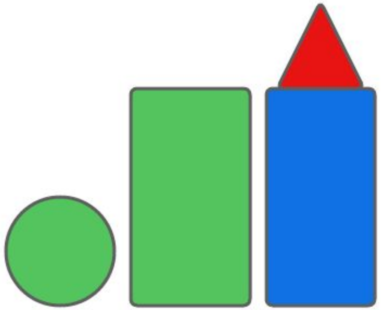
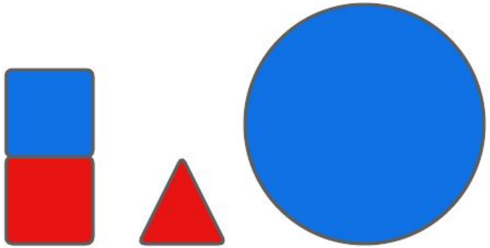
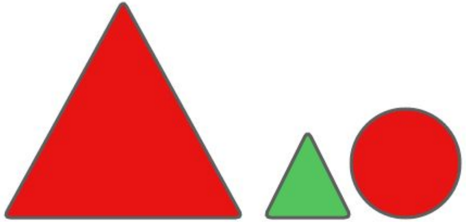
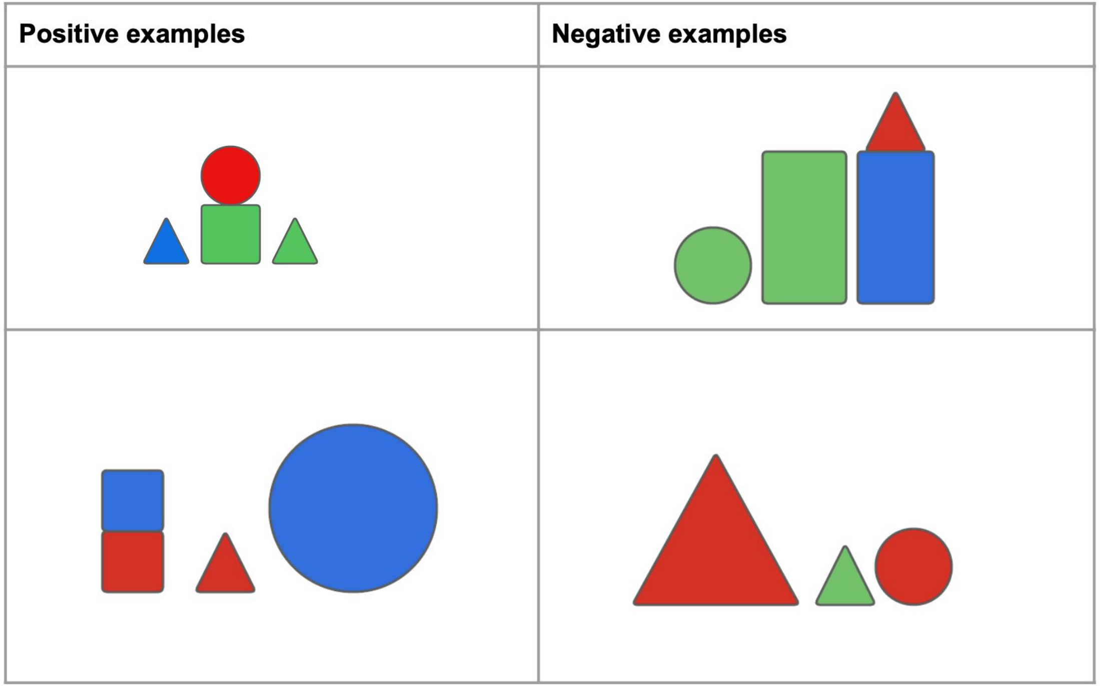
University of Oxford

UK Research and Innovation

UNIVERSITY OF OXFORD

Positive examples	Negative examples
	
	



There must be a red piece in contact with a small piece

Inductive Logic Programming (ILP)

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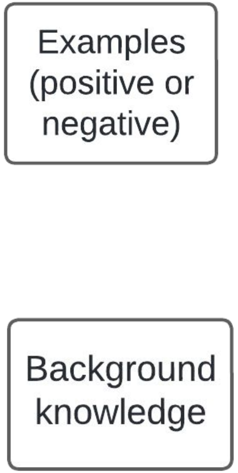
a form of program synthesis

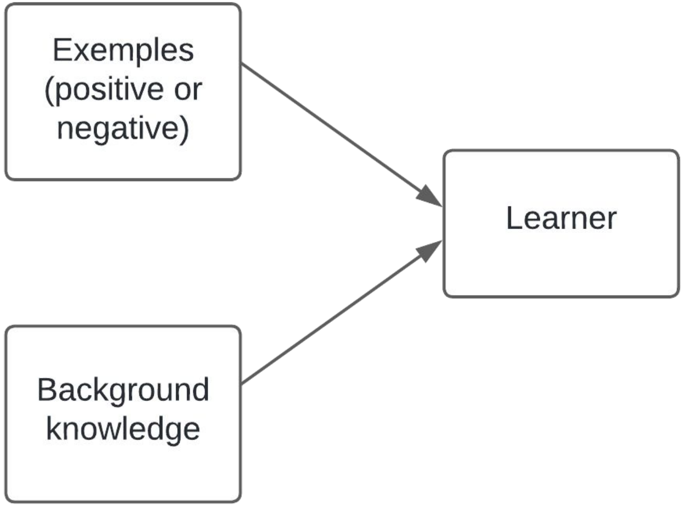
Inductive Logic Programming (ILP)

Examples (positive or negative)

Inductive Logic Programming (ILP)



Inductive Logic Programming (ILP)



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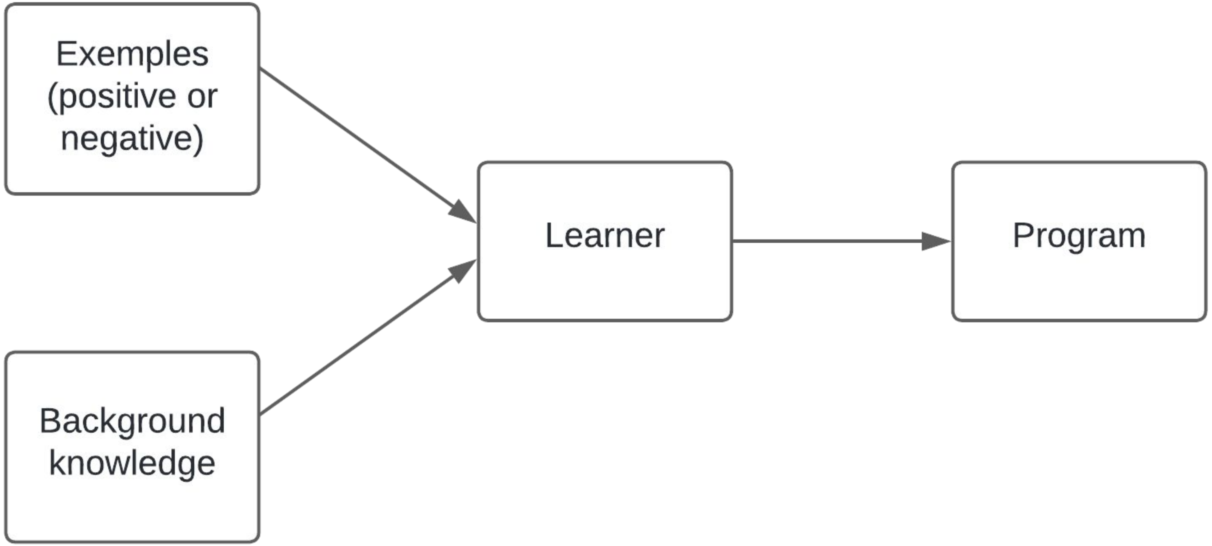

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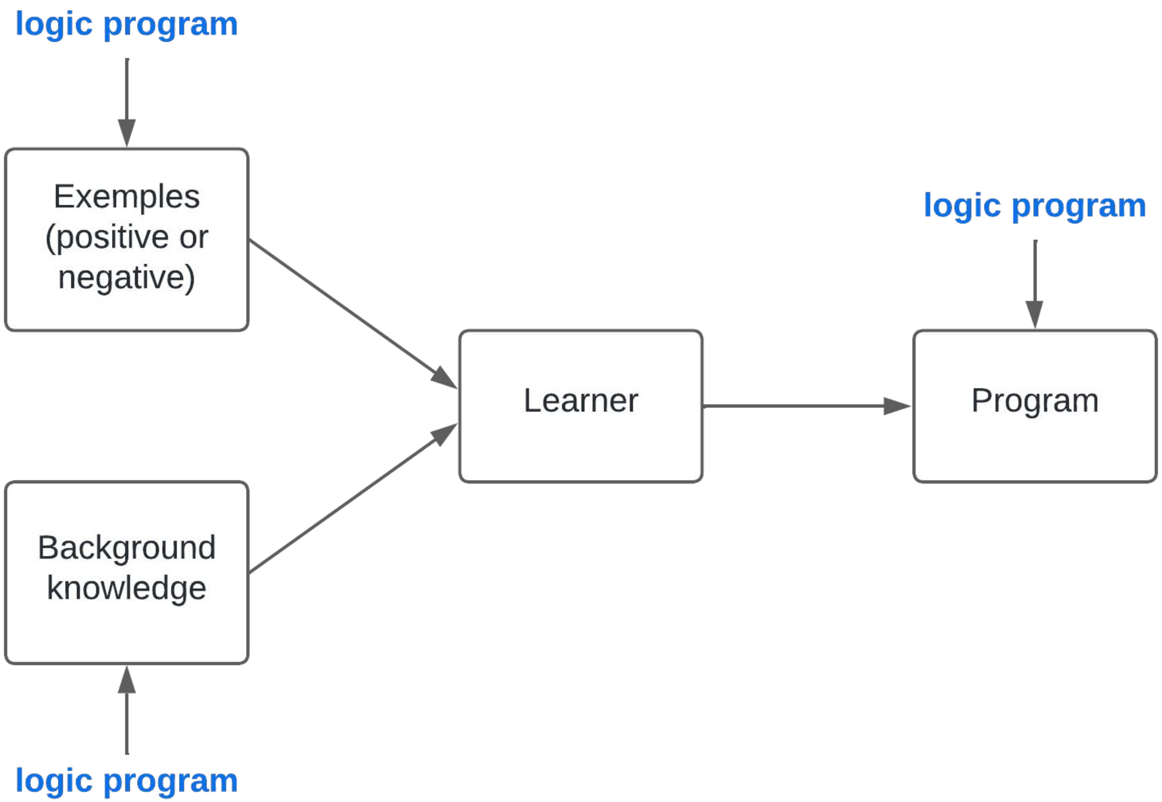
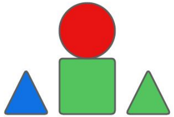
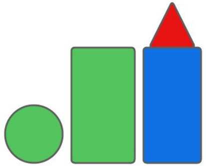


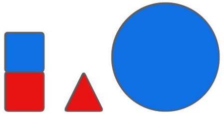
Positive examples	Negative examples
zendo(e_1). zendo(e_2).	zendo(e_{-1}). zendo(e_{-2}).



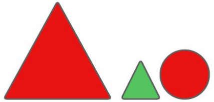
e_1



e_{-1}

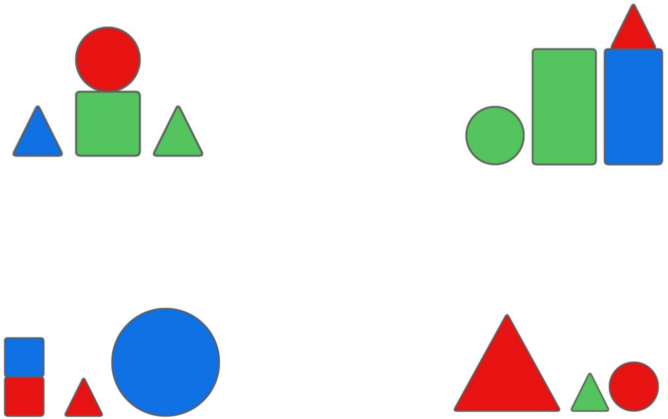


e_2



e_{-2}

Background Knowledge
piece(e_1, p1). piece(e_1, p2). piece(e_1, p3). piece(e_1, p4). blue(p1). triangle(p1). size(p1, 2). small(2). red(p2). round(p2). triangle(p4). contact(p2, p3). on(p2, p3). right(p4, p3). left(p1, p2). …



Program

```
zendo(Structure):-
    piece(Structure,Piece1),
    red(Piece1),
    contact(Piece1,Piece2),
    size(Piece2,Size),
    small(Size).
```

Popper: an inductive logic programming system

Why care?

Why care?

- learn globally optimal programs (textually minimal or minimal description length)

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- learn globally optimal programs (textually minimal or minimal description length)
- learn recursive programs
- support predicate invention
- learn large programs with many rules and large rules
- support noisy examples

How does it work?

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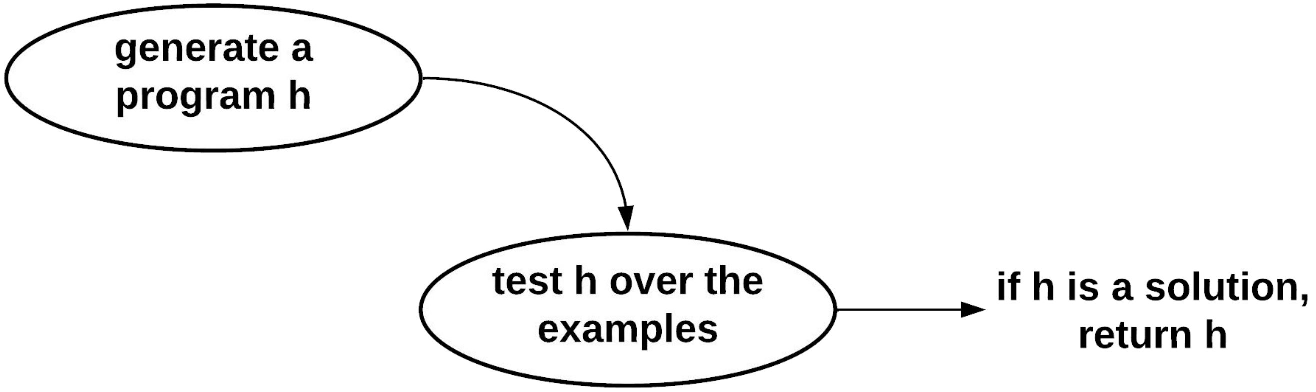

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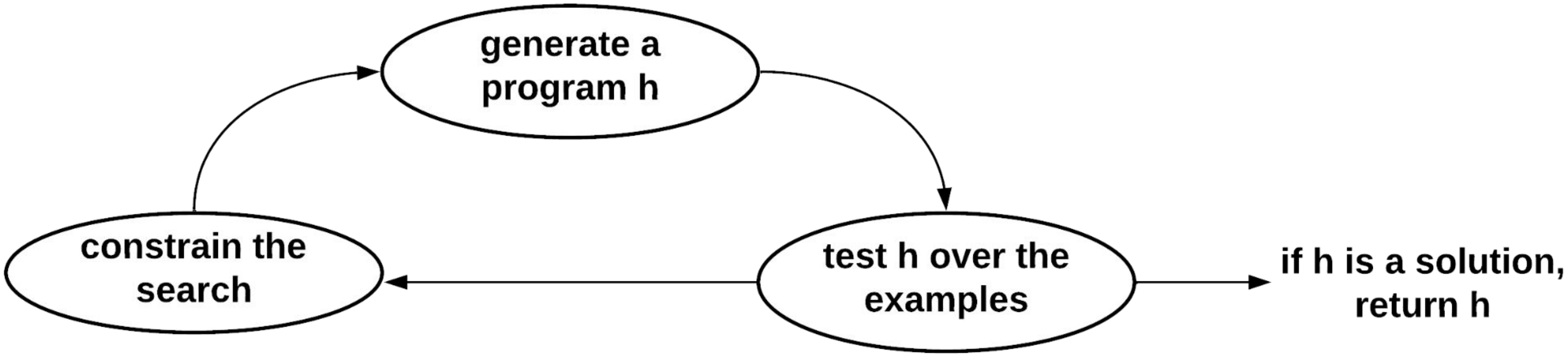
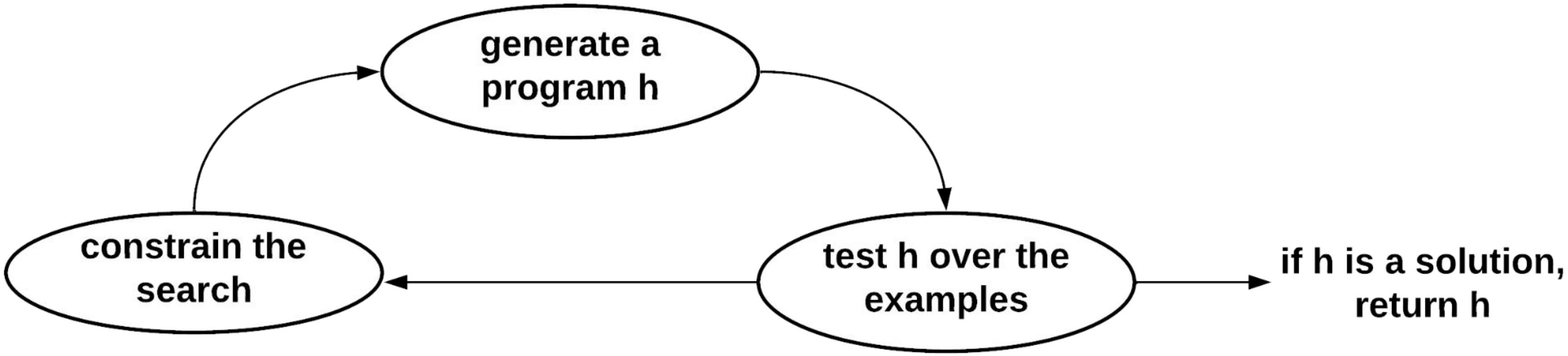


How does it work?

```
zendo(Structure):- piece(Structure,Piece),black(Piece).
```



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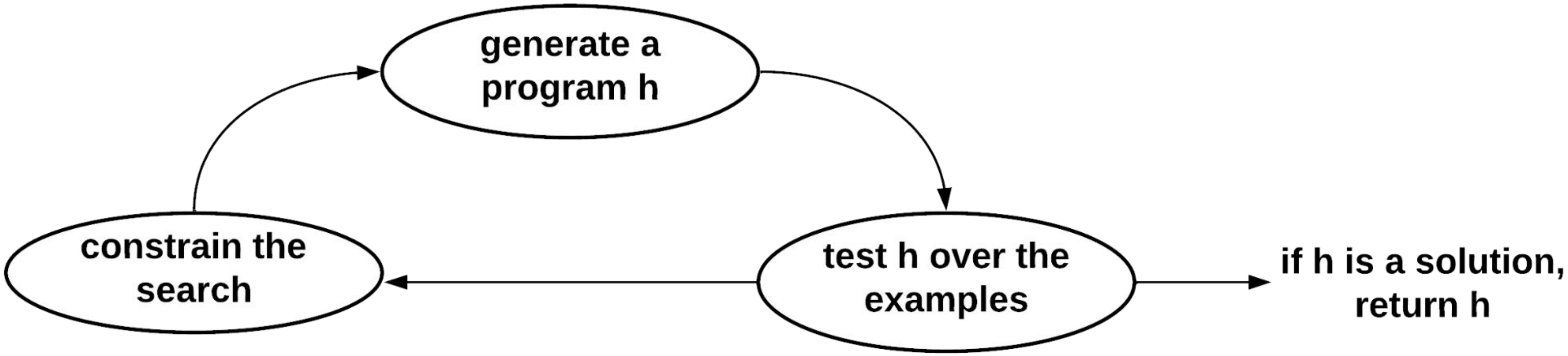

This program does not entail any positive example

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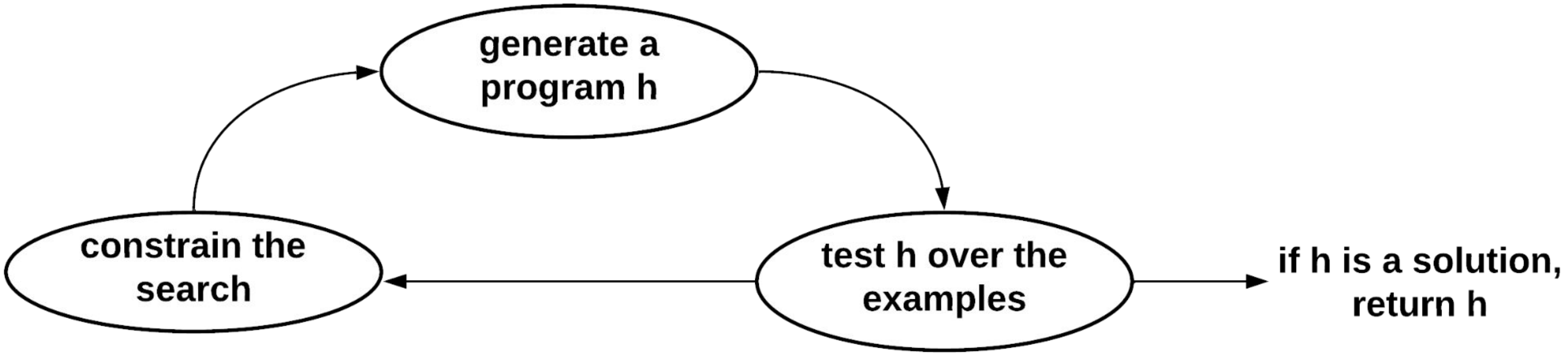


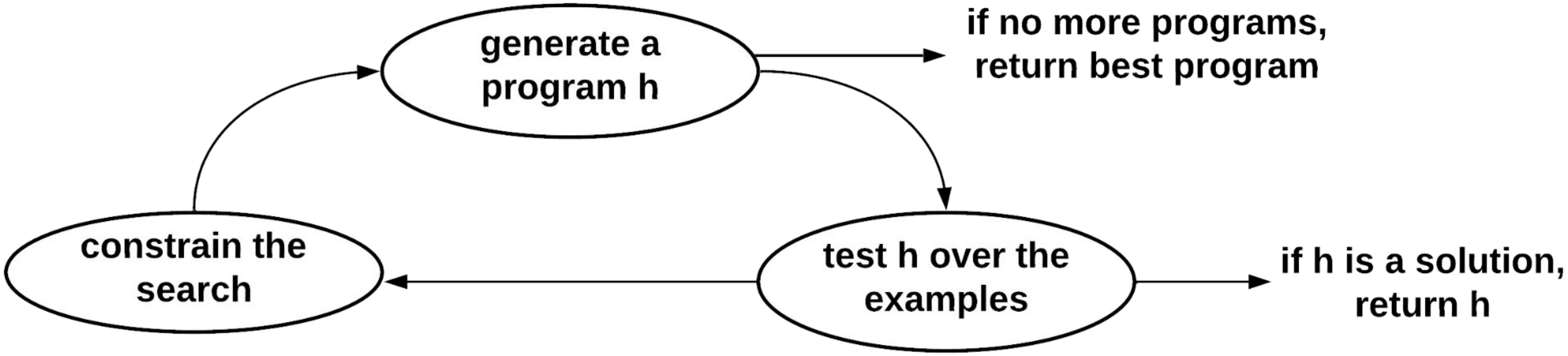
This program does not entail any positive example

We can prune its specialisations, such as:

```
zendo(Structure):- piece(Structure,Piece),black(Piece),contact(Piece,Piece1),blue(Piece1).
zendo(Structure):- piece(Structure,Piece),black(Piece),round(Piece).
zendo(Structure):- piece(Structure,Piece),black(Piece),size(Piece,Size),small(Size).
```

…

How does it work?



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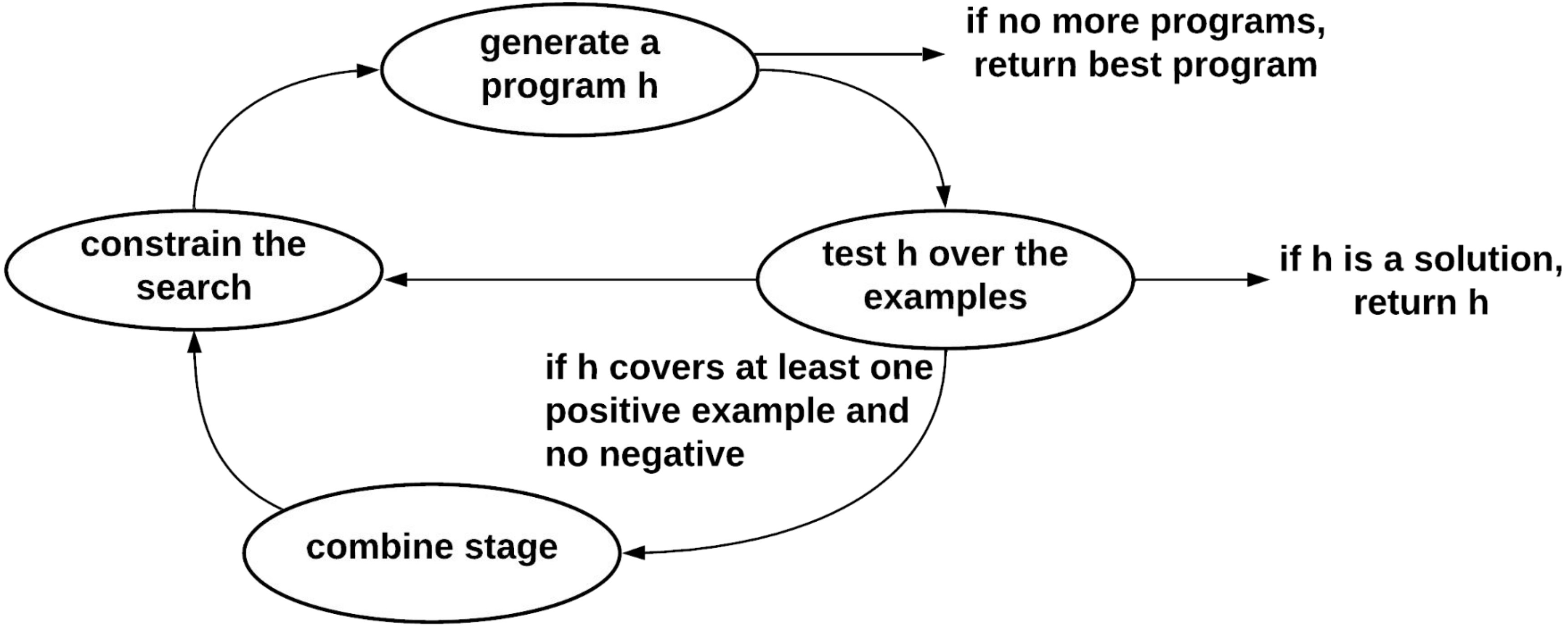


We use a MaxSAT solver to search for an optimal combination (a union) of programs.

Learning programs by combining programs, A. Cropper and C. Hocquette, *ECAI, 2023*.

Combine stage

```
p1: zendo(Structure):- piece(Structure,Piece),blue(Piece).                               covers {e1,e3,e7,e9} size 3
p2: zendo(Structure):- piece(Structure,Piece),yellow(Piece).                             covers {e2,e3} size 3
p3: zendo(Structure):- piece(Structure,Piece),red(Piece),square(Piece).               covers {e2,e4,e6} size 4
p4: zendo(Structure):- piece(Structure,Piece1),contact(Piece1,Piece2),yellow(Piece2).   covers {e5,e8,e9} size 4
p5: zendo(Structure):- piece(Structure,Piece),size(Piece,Size),small(Size).             covers {e7,e8,e9} size 4
p6: zendo(Structure):- piece(Structure,Piece1),blue(Piece1),piece(Structure,Piece2),red(Piece2).  covers {e5} size 5
p7: zendo(Structure):- piece(Structure,Piece),green(Piece),size(Piece,Size),large(Size).          covers {e4,e5} size 5
p8: zendo(Structure):- piece(Structure,Piece),contact(Piece1,Piece2),red(Piece2),square(Piece2).     covers {e6,e7} size 5
p9: zendo(Structure):- piece(Structure,Piece),red(Piece1),contact(Piece1,Piece2),blue(Piece2),round(Piece2). covers {e8} size 6
…
```

Combine stage

p1: zendo(Structure):- piece(Structure,Piece),blue(Piece). covers {e_1,e_3,e_7,e_9} size 3

~~p2: zendo(Structure):- piece(Structure,Piece),yellow(Piece). covers {e_2,e_3} size 3~~

p3: zendo(Structure):- piece(Structure,Piece),red(Piece),square(Piece). covers {e_2,e_4,e_6} size 4

p4: zendo(Structure):- piece(Structure,Piece1),contact(Piece1,Piece2),yellow(Piece2). covers {e_5,e_8,e_9} size 4

~~p5: zendo(Structure):- piece(Structure,Piece),size(Piece,Size),small(Size). covers {e_7,e_8,e_9} size 4~~

~~p6: zendo(Structure):- piece(Structure,Piece1),blue(Piece1),piece(Structure,Piece2),red(Piece2). covers {e_5} size 5~~

~~p7: zendo(Structure):- piece(Structure,Piece),green(Piece),size(Piece,Size),large(Size). covers {e_4,e_5} size 5~~

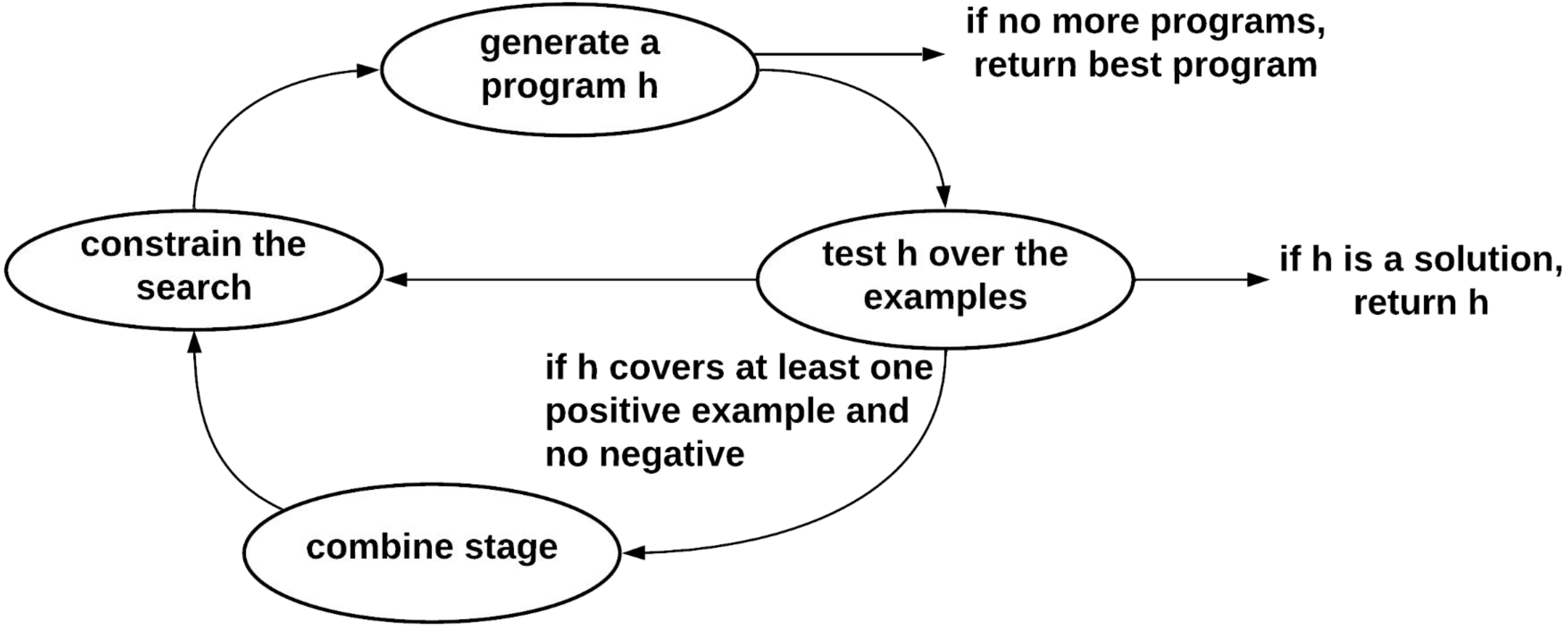
~~p8: zendo(Structure):- piece(Structure,Piece),contact(Piece1,Piece2),red(Piece2),square(Piece2). covers {e_6,e_7} size 5~~

~~p9: zendo(Structure):- piece(Structure,Piece),red(Piece1),contact(Piece1,Piece2),blue(Piece2),round(Piece2). covers {e_8} size 6~~

…

{p1, p3, p4} entails all the positive examples and has minimal size

How does it work?



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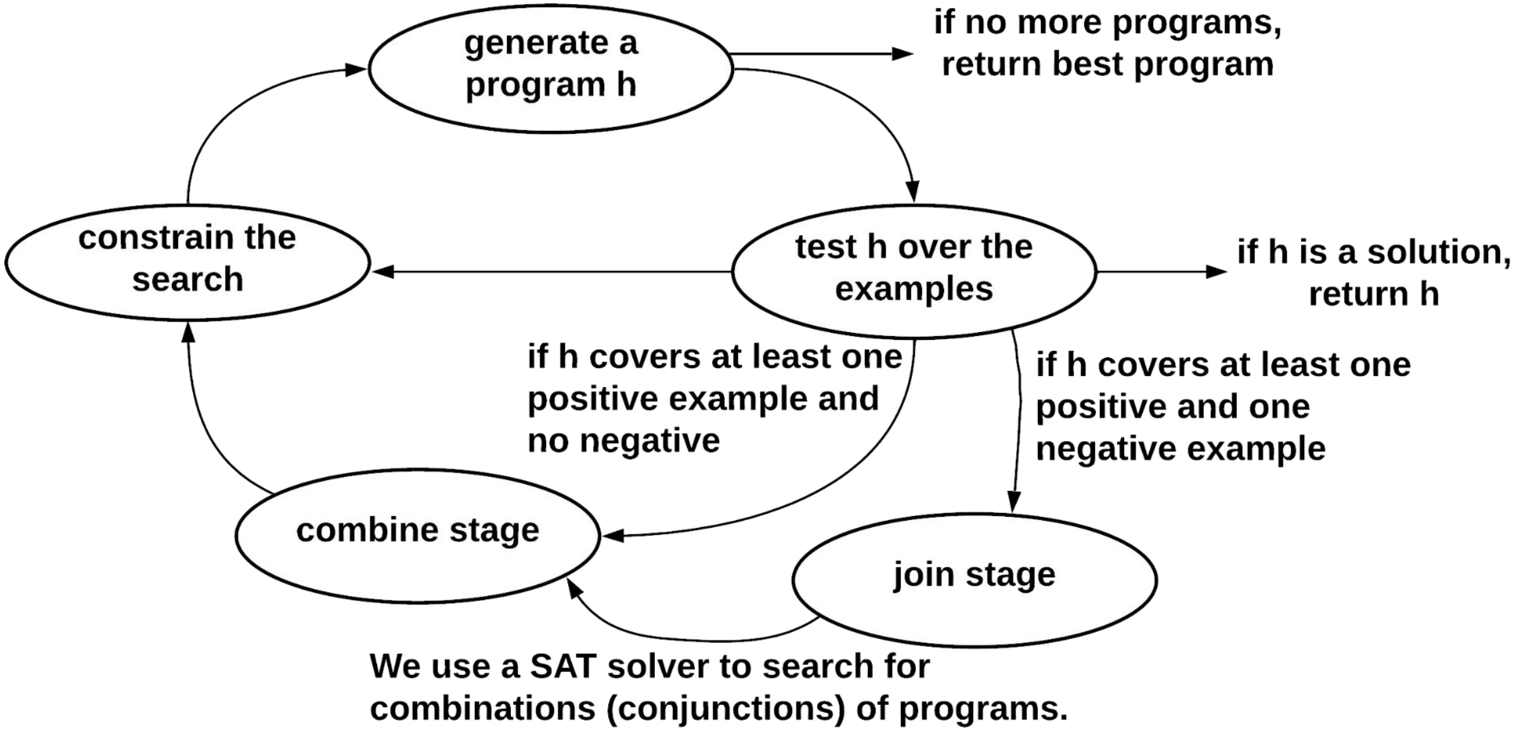

Learning big logical rules by joining small rules, C. Hocquette, A. Niskanen, R. Morel, M. Järvisalo, and A. Cropper, *AAAI, 2023*.

Join stage

```
p1: zendo(Structure):- piece(Structure,Piece),blue(Piece).
p2: zendo(Structure):- piece(Structure,Piece),yellow(Piece).
p3: zendo(Structure):- piece(Structure,Piece),red(Piece),square(Piece).
p4: zendo(Structure):- piece(Structure,Piece),contact(Piece,Piece1),blue(Piece1).
…
```

covers $\{e_1,e_3,e_7,e_9\}$ $\{e_{-1},e_{-2}\}$ size 3

covers $\{e_1,e_3,e_7,e_9\}$ $\{e_{-4},e_{-5}\}$ size 3

covers $\{e_1,e_2,e_3,e_4,e_6\}$ $\{e_{-6}\}$ size 4

covers $\{e_7,e_8\}$ $\{e_{-6}\}$ size 4

Join stage

```
p1: zendo(Structure):- piece(Structure,Piece),blue(Piece).
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…
```

p1 covers $\{\mathbf{e_1},\mathbf{e_3},e_7,e_9\}$ $\{e_{-1},e_{-2}\}$ size 3

p2 covers $\{\mathbf{e_1},\mathbf{e_3},e_7,e_9\}$ $\{e_{-4},e_{-5}\}$ size 3

p3 covers $\{\mathbf{e_1},e_2,\mathbf{e_3},e_4,e_6\}$ $\{e_{-6}\}$ size 4

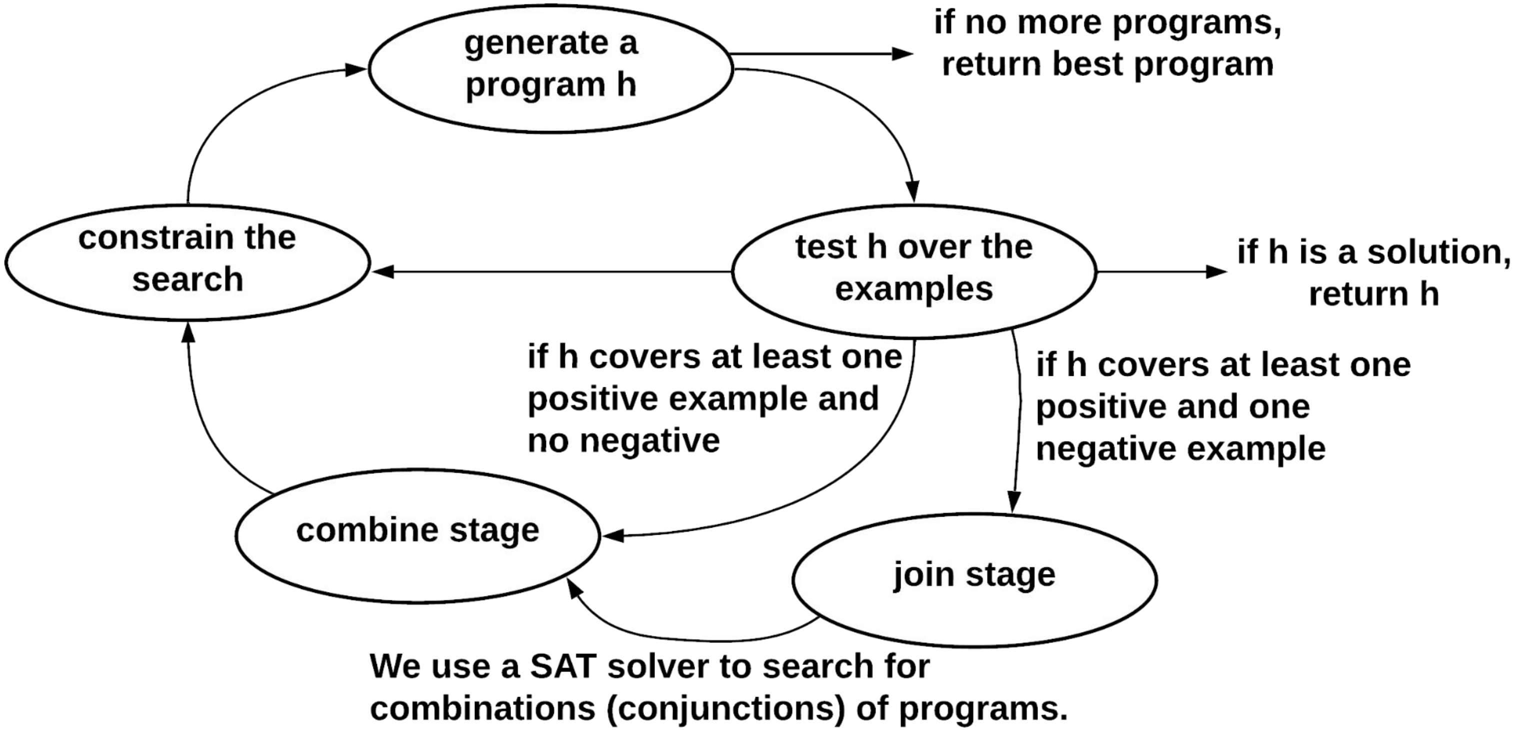
p4 covers $\{e_7,e_8\}$ $\{e_{-6}\}$ size 4

$p = p_1 \cap p_2 \cap p_3$

```
p: zendo(Structure):- piece(Structure,Piece),blue(Piece),
                      piece(Structure,Piece),yellow(Piece),
                      piece(Structure,Piece),red(Piece),square(Piece).
```

p covers $\{e_1,e_3\}$ and no negative example

How does it work?



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Correctness

Popper learns an optimal solution (a textually minimal program).

Learning from noisy data

Learning from noisy data

minimum description length: trade-off model complexity (program size) and data fit (training accuracy)

$$mdl(h) = size(h) + fp(h) + fn(h)$$

Learning MDL Logic Programs From Noisy Data, C. Hocquette, A. Niskanen, M. Järvisalo, A. Cropper, AAAI24.

https://github.com/logic-and-learning-lab/Popper

Conclusion

- Popper, an ILP algorithm

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    - feature-rich:
        - recursive
        - predicate invention
        - optimal programs (mdl or textually minimal)
        - noisy data
        - anytime
        - infinite domains and numerical reasoning

Conclusion

- Popper, an ILP algorithm
    - feature-rich:
        - recursive
        - predicate invention
        - optimal programs (mdl or textually minimal)
        - noisy data
        - anytime
        - infinite domains and numerical reasoning
    - can learn moderately large programs (largish rules and many rules)

Limitations

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- Very large datasets with lots of BK and lots of examples (10k+)

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- Learn rules with many variables (long-chains of reasoning)
- Invent complex abstractions

Tips

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- if possible, use datalog BK
- avoid recursion if possible
- avoid predicate invention if possible
- use a sat solver for the combine stage

Thank you!